DISTANCE ESTIMATION BASED ON LIGHT FIELD GEOMETRIC MODELING

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ABSTRACT
In this paper, geometric optical models are proposed that can accurately estimate the distances of object planes in a light field image. Treating an arbitrary point on an object plane as an off-axis point light source, the single-point-light-source model, which consists of two relaying models, is established to describe the propagation of light rays from the off-axis point light sources to the image sensor. Based on the relaying models, the distances of object planes can be estimated by deriving the relationship between the positions of light rays impinging on the image sensor and incident angles of light rays entering into the main lens. Furthermore, taking into account the realistic imaging, the so-called line-source model is established by extending the off-axis point light sources to general line sources. The performances of the proposed models are compared with existing distance estimation methods under two optical configurations of plenoptic 1.0 cameras and results demonstrate that the proposed methods improve estimation accuracy by an average of 2.315%/4.675% for point light sources and 2.77% for a line source.

Index Terms—Light field, distance estimation, off-axis point light source, relaying models, line source

1. INTRODUCTION
Light field imaging becomes increasingly attractive in a variety of applications [1-3]. Compared with other light field acquisition systems [4-6], microlens array (MLA) based hand-held light field cameras offer a cost-effective approach, and have been adopted into industry and consumer market, known as plenoptic 1.0 (Lytro) and 2.0 (Raytrix) cameras [7-8]. Different from traditional cameras, plenoptic cameras can capture both spatial and angular light information (using plenoptic 1.0 cameras as an instance in Fig.1), which enable new capabilities, such as refocusing [9], viewpoint synthesis [10], and extending the depth of field after capturing [11]. Currently, estimating the distances of object planes in an image captured by a plenoptic 1.0 camera is becoming another attractive topic to its applications.

Distance can be inversely calculated by utilizing the depth information estimated from the light field [12-13]. However, the depth maps are usually rough due to the narrow baseline and low spatial resolution of plenoptic 1.0 cameras, especially in texture-less regions. Thus, the accuracy is too limited. Hanne et al. [14] proposed a ray tracing method based on the imaging system of plenoptic 1.0 cameras, which can estimate the distances of object planes the user is refocusing. Nevertheless, the method is incapable to estimate the distances of all object planes through only one refocusing implementation. In order to solve this drawback and simultaneously provide higher estimation accuracy, Chen et al. [15] proposed a geometric optical model which estimates the distance by investigating its relationship with the imaging diameter. However, the model provided in [15] is derived on account of on-axis point light sources. Therefore, it is lack of generality and practicability.

Fig.1. Plenoptic 1.0 cameras.
especially at general imaging range.

The rest of the paper is organized as follows. Section 2 describes the geometric optical models in detail based on the optical analysis of single point light source imaging and a general line source imaging. Section 3 provides performance comparison and analyses. Conclusions are drawn in Section 4 as well as some potential future works.

2. GEOMETRIC OPTICAL MODELS

2.1. Optical Analysis

For an arbitrary point on the object plane, light rays emitting from the point can be considered as propagating from an off-axis point light source with an offset \( \Delta h \), as shown in Fig.2. Thus, an on-axis source corresponds to \( \Delta h = 0 \). With the assumptions that light rays entering the main lens by covering its whole pupil diameter and refractions on MLA are neglected, imaging of a point light source varies with the distance of the plane which the point light source is located on, as depicted in Fig.2. As we can see from Fig.2, the imaging diameters on the image sensor are different and dependent on the defocuses of the object planes, i.e. \( a, i \) and \( d \), which directly relates to the distance between the object plane and the main lens. This discovery implies that the distance of an object plane is possible to be estimated by investigating its association with the imaging diameter.

![Fig.2. Imaging of off-axis point light sources at different distances. The offset apart from the optical axis is \( \Delta h \).](image1)

2.2. Single-Point-Light-Source Geometric Optical Model

According to the above analysis, a single-point-light-source model is proposed, which consists of two relaying models, namely object space model and image space model.

Using plane \( i \) in Fig.2 as an instance, the object space model is displayed in Fig.3. In this relaying model, light rays emitting from an off-axis point light source with offset being \( \Delta h \), propagate into the main lens and refract inside it following refraction theorem. As shown in Fig.3, \( d_{out}' \) defines the distance between plane \( i \) and the main lens, which is exactly needed to be estimated. Its relationship with the incident angle, denoted by \( \varphi \), in Fig.3, is mathematically given by

\[
\tan \varphi = \frac{D/2 + (\text{sgn}(i)\Delta h)}{(d_{out}' - T/2 + R - \sqrt{R^2 - D^2}/4)}, \tag{1}
\]

where \( R \) represents the radius of curvature of main lens; \( T \) represents the central thickness of main lens; \( D \) is the pupil diameter of main lens; \( \text{sgn}(i) \) is defined by

\[
\text{sgn}(i) = \begin{cases} -1 & i = 1 \\ 1 & i = 2 \end{cases} \tag{2}
\]

Eq.(1) indicates that \( \Delta h \) and \( d_{out}' \) can be obtained by calculating

\[
\Delta h = \frac{D(\tan \varphi_2 - \tan \varphi_1)}{2(\tan \varphi_1 + \tan \varphi_2)}, \tag{3}
\]

\[
d_{out}' = \frac{D/2 + (\text{sgn}(i)\Delta h)}{\tan \varphi_1} + \sqrt{R^2 - D^2}/4 - R + T/2.
\]

Eq.(3) implies that \( \varphi_1 \) is needed to be obtained in order to calculate \( \Delta h \) and further estimate \( d_{out}' \).

![Fig.3. Object space model: light rays propagation from the point light source to the main lens.](image2)

After entering the main lens, light rays will refract inside the main lens and respectively arrive at \((p_1, q_1)\), which is marked by green dot. The refractive angle, \( \psi_i \), satisfies the refraction theorem, which is given by

\[
n_i \sin \psi_i = \sin(\varphi_1 + \theta), \tag{4}\]

where \( n_i \) is the refractive index of the main lens; \( \psi_i \) is the included angle between the normal (the dashed lines highlighted in purple) and the refractive light rays inside the main lens; \( \theta \) follows

\[
\sin \theta = \frac{D}{2R}. \tag{5}\]

After arriving at \((p_1, q_1)\) as marked in Fig.3, light rays will exit from these positions with refractive angles \( \omega_1 \), and then impinge on the image sensor. These propagations of light rays are carried out in the image space and the proposed image space model is shown in Fig.4.

By utilizing the refraction theorem again, it turns out that

\[
n_i \sin(\theta - \psi_i + \phi) = \sin \omega_1, \tag{6}\]

where \( \phi \) follows

\[
\sin \phi = \frac{|q_1|}{R}. \tag{7}\]
As the refractions on MLA are ignored, we have
\[
\tan(\omega_i - \phi_i) = \frac{q_i + (\text{sgn}(i)\hat{v}_i)}{f_s + d_{in} - p_i}, \tag{8}
\]
where \(\hat{v}_i\) represents the vertical coordinates and \(|\hat{v}_i - \hat{v}_j|\) equals to the imaging diameter of light rays on the image sensor; \(f_s\) is the focal length of MLA and \(d_{in}\) defines the distance between MLA and main lens. Besides, \((p_i, q_i)\) lies on the curved surface of main lens so that \(p_i\) and \(q_i\) satisfy
\[
(R - T/2 + p_i)^2 + q_i^2 = R^2, \tag{9}
\]
Analyses on Eqs.(8) and (9) manifest that \(p_i\) and \(q_i\) can only be obtained after \(d_{in}\), \(\hat{v}_i\), and \(\omega_i - \phi_i\) are all known.

In order to get the values of \(d_{in}\), \(\hat{v}_i\), refocusing and inverse ray tracing are required. Therefore, the light field image is refocused to a reference plane at a known distance away from the main lens once it is captured. The refocusing is implemented by using the synthesis technique proposed in [14] as shown in Fig.5. The reference plane can be identified as the plane \(\alpha\) in Fig.2 whose distance is \(d_{out}\). The method [14] traces a pair of light rays emitting from corresponding pixels on the image sensor back to the object space, and the intersection of light rays with the same color is taken as the distance of the refocused object plane. Therefore, the slopes of light rays in image space, \(m_i\), and object space, \(s_{ij}\), are required to be obtained, which are respectively given by
\[
m_i = \frac{y_j - v_{ik}}{f_s}, \tag{10}
\]
\[
s_{ij} = \frac{y_j' - F_i}{d_{out} - f}, \tag{11}
\]
where \(y_j\) represents the vertical central coordinates of each micro lens and \(v_{ij}\) represents the vertical central coordinates of each pixel on the image sensor; \(y_j'\) denotes the known vertical coordinates of calibrated points on plane \(\alpha\); \(f\) is the focal length of main lens; \(F_i\) denotes the interval between the intersections of corresponding light rays on plane \(F_s\), and the intervals are the baselines of virtual viewpoints in hand-held plenoptic 1.0 cameras [14].

\[\hat{v}_i\] can then be achieved by recording the vertical coordinates of light rays that impinge on the image sensor, in the refocused light field image.

For the purpose of achieving the value of \(\omega_i - \phi_i\), an approximation is made in the image space model that light rays will intersect at the yellow dots as marked in Fig.4, the centers of marginal pupil diameter of main lens, supposing the light rays are prolonged on the main lens. By combining Eq.(8) and utilizing similar triangle principle which gives
\[
\frac{D}{2 + (\text{sgn}(i)\hat{v}_i)} \approx \frac{q_i + (\text{sgn}(i)\hat{v}_i)}{f_s + d_{in} - p_i}, \tag{13}
\]
\(\omega_i - \phi_i\) can then be approximately achieved by
\[
\tan(\omega_i - \phi_i) \approx D/2 + (\text{sgn}(i)\hat{v}_i) \frac{f_s + d_{in} - p_i}{f_s + d_{in}}. \tag{14}
\]

After the above processing and approximation, \(p_i\) and \(q_i\) can be derived by plugging \(d_{in}\), \(\hat{v}_i\), and \(\omega_i - \phi_i\) into Eqs.(8) and (9), which is used for deriving \(d_{out}\).

After that, refractive angle \(\omega_i\) can be calculated by
\[
\omega_i = \arctan\left(\frac{q_i + (\text{sgn}(i)\hat{v}_i)}{f_s + d_{in} - p_i} + \arcsin\frac{|q_i|}{R}\right). \tag{15}
\]

Subsequently, \(\psi_i\) and \(\phi_i\) can be obtained by solving
\[
\psi_i = \arcsin\left(\frac{D}{2R}\right) + \arcsin\left(\frac{|q_i|}{R}\right) - \arcsin\left(\frac{|q_i|}{R}\right), \tag{16}
\]
\[
\phi_i = \arcsin(n_i \sin(\psi_i)) - \arcsin\left(\frac{D}{2R}\right). \tag{17}
\]
Finally, $d_{\text{out}}$ can be obtained by plugging the calculated $\varphi_i$ into Eq.(3).

For plane $\tilde{r}$ in Fig.2, its object space model is the same as Fig.3 shows. Difference exists in the image space model since currently the focal plane in image space is located between the main lens and MLA. However, deriving the distance of plane $\tilde{r}$ is exactly the same as plane $\tilde{r}$.

2.3. Line-Source Geometric Optical Model

Considering that the realistic imaging is more complicated, the above model derived based on an off-axis point light source is extended to a general line source. A line source can be regarded as a combination of infinite point light sources. For the $i$th point light source on the line, Eq.(3) indicates that

$$D/2 - \Delta h_i = d \tan \varphi_i, \quad i \in [1, n]$$

where $d = d'_{\text{out}} - \sqrt{R^2 - D^2}/4 - R + T/2$ and $\varphi_i$ denotes the angle $\varphi_i$ of the $i$th point light source. Note that $\varphi_{i+1} = \frac{\Delta h_i}{\tan \varphi_{i+1} - \tan \varphi_i}$ can also be used to derive an equation like (18) while $D/2 - \Delta h_i$ should be replaced by $D/2 + \Delta h_i$. Therefore, we have

$$d_i \approx \frac{\Delta h_i - \Delta h_{i-1}}{\tan \varphi_{i+1} - \tan \varphi_i}, \quad i \in [1, n].$$

Eq.(20) demonstrates that for a line source, the distance between the plane, where the line source is located, and the main lens can be estimated by using the imaging information of two endpoints of the line source. It is applicable since the imaging diameter of a line source inherently is determined by the two endpoints of the line source, as shown in Fig.6.

Fig.6. Imaging of line sources at different distances.

Utilizing the fact that the imaging diameter of a point light source on the same object plane is almost the same, $\tilde{v}_i$ and $\tilde{v}_2$ for two endpoint light sources in Fig.6 can be obtained, respectively. After that, $\Delta h_i$ and $\Delta h_{i-1}$ can be estimated using the derivations described in Section 2.2 and the distance of the object plane can be estimated using Eq.(20).

3. PERFORMANCE COMPARISON AND ANALYSES

3.1. Simulation System

In order to validate the effectiveness of the proposed geometric optical models, imaging systems of hand-held plenoptic 1.0 cameras are simulated in optics tool Zemax [16], as shown in Fig.7. The gap between the image sensor and MLA is magnified for better visibility. The performances of the proposed models are compared with the method provided in [14]. Two general optical settings of plenoptic 1.0 cameras designed in [14] and [17] are used for the simulated system 1 and 2, respectively. The optical parameters of each system are listed in Table.1. The parameters of main lens in system 2 are changed a bit to keep its $F$–number being equal to that of the MLA.

Fig.7. Zemax screenshots: (a) Simulated imaging system of hand-held plenoptic 1.0 cameras; (b) Zoomed MLA.

Table.1. Parameters of two simulated imaging systems.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Imaging system 1</th>
<th>Imaging system 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal length of each micro lens, $f_x$</td>
<td>2.749mm</td>
<td>0.7mm</td>
</tr>
<tr>
<td>Pitch size of each micro lens, $D_x$</td>
<td>1.34mm</td>
<td>0.25mm</td>
</tr>
<tr>
<td>Focal length of main lens, $f$</td>
<td>99.515mm</td>
<td>98.088mm</td>
</tr>
<tr>
<td>Pupil diameter of main lens, $d$</td>
<td>50mm</td>
<td>40mm</td>
</tr>
<tr>
<td>Central thickness of main lens, $T$</td>
<td>14.35mm</td>
<td>6mm</td>
</tr>
<tr>
<td>Radius of curvature of main lens, $R$</td>
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<td>100mm</td>
</tr>
<tr>
<td>Refractive index of main lens, $n_i$</td>
<td>1.515</td>
<td>1.515</td>
</tr>
<tr>
<td>Wavelength of light rays, $\lambda$</td>
<td>632.8nm</td>
<td>632.8nm</td>
</tr>
</tbody>
</table>

3.2. Results and Analyses

The estimation error of distance, denoted by $ERROR$, is used for comparing the performances of the proposed models and the method in [14], which is given by

$$ERROR = \left| \frac{d'_{\text{out}} - Ed'_{\text{out}}}{Ad + T/2} - \frac{Ad + T/2}{Ad + T/2} \right|$$

where $d'_{\text{out}}$ represents the real distances of object planes, such as plane $\tilde{r}$ and $\tilde{r}$, as depicted in Fig.2; $Ed'_{\text{out}}$ denotes
the estimated distances of object planes that are achieved according to the derivations in Section 2; $Ad$ refers to the actual distances between object planes and the vertex of right curved surface of main lens on the axis. Therefore, $d_{out}$ equals to $Ad+T/2$; $Ed$ represents the estimated distances between object planes and the vertex of right curved surface of main lens on the axis, which means $Ed_{out}$ equals to $Ad+T/2$. Results of estimated distances and estimation errors using single point light source with different $\Delta h$ in the two imaging systems are listed in Table.2. Note that estimating the distance using the method in [14] is not associated with $\Delta h$ nor the type of light sources.

### Table.2. Estimation error comparison for an off-axis point light source.

<table>
<thead>
<tr>
<th>$Ad$ (mm)</th>
<th>Imaging system 1</th>
<th>Imaging system 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta h = 5mm$</td>
<td>$\Delta h = 10mm$</td>
</tr>
<tr>
<td>$Ed$ (mm)</td>
<td>ERROR (%)</td>
<td>$Ed$ (mm)</td>
</tr>
<tr>
<td>-----------</td>
<td>------------</td>
<td>-----------</td>
</tr>
<tr>
<td>3500</td>
<td>3278.40</td>
<td>6.32</td>
</tr>
<tr>
<td>3250</td>
<td>3020.35</td>
<td>7.05</td>
</tr>
<tr>
<td>3000</td>
<td>2792.09</td>
<td>6.91</td>
</tr>
<tr>
<td>2800</td>
<td>2586.75</td>
<td>7.24</td>
</tr>
<tr>
<td>2500</td>
<td>2333.13</td>
<td>6.66</td>
</tr>
<tr>
<td>2200</td>
<td>2047.86</td>
<td>6.89</td>
</tr>
<tr>
<td>2000</td>
<td>1856.75</td>
<td>7.14</td>
</tr>
<tr>
<td>1800</td>
<td>1665.32</td>
<td>7.45</td>
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<td>1500</td>
<td>1387.31</td>
<td>7.48</td>
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<td>1300</td>
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<td>1100</td>
<td>1018.54</td>
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<td>900</td>
<td>833.74</td>
<td>7.30</td>
</tr>
<tr>
<td>750</td>
<td>695.43</td>
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</tr>
<tr>
<td>600</td>
<td>557.19</td>
<td>7.05</td>
</tr>
<tr>
<td>450</td>
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<td>6.73</td>
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<tr>
<td>300</td>
<td>281.76</td>
<td>5.94</td>
</tr>
<tr>
<td>Ave. (%)</td>
<td>7.30</td>
<td>5.04</td>
</tr>
</tbody>
</table>

### Table.3. Estimation error comparison for a line source.

As shown in Table.2, the proposed geometric optical model outperforms the method in [14] for the majority of distances in imaging system 1 and the whole distance range in imaging system 2. In addition, the proposed model is superior to the method [14] by an average of 2.26%/2.37% reduction in estimation error as $\Delta h = 5mm$ / $\Delta h = 10mm$ in imaging system 1. In imaging system 2, the average of estimation error reduces almost 3 times, and the maximum reduction reaches 55 times at 0.3m, which implies high potential of the proposed model in applications. It is also found that the obtained estimation errors of the two imaging systems are different, which indicates that the optical parameters of plenoptic 1.0 cameras can influence the estimation accuracy. The effects of changing parameters are under investigations as one of our future works.

Results of estimated distances and estimation errors using a 10mm line source in imaging system 1 are listed in Table.3. Results show that the proposed model can provide obvious improvement in the estimation accuracy, an average of 2.77% reduction compared with that of [14], at most of the distances, which demonstrates the effectiveness of the generalization.

The estimation error results listed in Table.2 and Table.3 are graphed in Fig.8. It is observed that the performances of the proposed models are slightly worse than [14] at quite farther distances in imaging system 1. As pointed out in [15], the reason mainly lies in the approximation made in the image space model. During deriving Eq.(14), light rays $r_1$ and $r_2$ are respectively approximated by $r'_1$ and $r'_2$, as shown in Fig.9. Thus, deviations exist in the light rays emission position $(p, q)$ on the main lens, which leads to the estimation errors. Farther distances on the object space generally cause larger $\Delta q$, and finally result in larger errors $Ed+T/2$. Results of estimated distances and estimation errors using single point light source with different $\Delta h$ in the two imaging systems are listed in Table.2. Note that estimating the distance using the method in [14] is not associated with $\Delta h$ nor the type of light sources.
estimation errors. However, the errors can be compensated by changing the parameters of plenoptic 1.0 cameras, as referred to the results of the two imaging systems in Table.2.

![Results of imaging system 1 in Table.2](image1)

![Results of imaging system 2 in Table.2](image2)

![Results of the line source in Table.3](image3)

**Fig.8.** Estimation error comparison.

**Fig.9.** Ray tracing model for estimation error analysis.

### 4. CONCLUSIONS

In this paper, geometric optical models are put forward to estimate the distances of object planes in a light field image. Experimental results demonstrate that the proposed models can outperform existing distance estimation methods in terms of accuracy, particularly in general imaging range. We plan to build a real imaging system to verify the effectiveness of the proposed models while some problems need to be considered. For example, the noises, like calibration errors, should be compensated by introducing some correction factors into the proposed models. Besides, a simple thin lens is actually used in the proposed models for the sake of light weight and low cost of the plenoptic camera whereas includes optical aberrations, so it is important for us to correct the aberrations to restore light field images with high quality. We also imagine to extend the work to more areas, such as underwater imaging and microscopy.

### 5. ACKNOWLEDGEMENT

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### 6. REFERENCES


